بسم الله الرحمن الرحيم

Benha University
Shoubra Faculty of Engineering

Mechanical Department
1st year Mech.

10.03.2019 – Week 5

Bending test

Prof. Farida Sayed Ahmed
Dr. Mahmoud Khedr

Outline

- Stress distribution over the cross section under tension/compression.
- Theory of simple bending.
- Beam bending test.
- The mechanical properties after bending test.
- Cold Bent Test
- Transverse shear stress

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Stress distribution over the cross section under tension/compression loading

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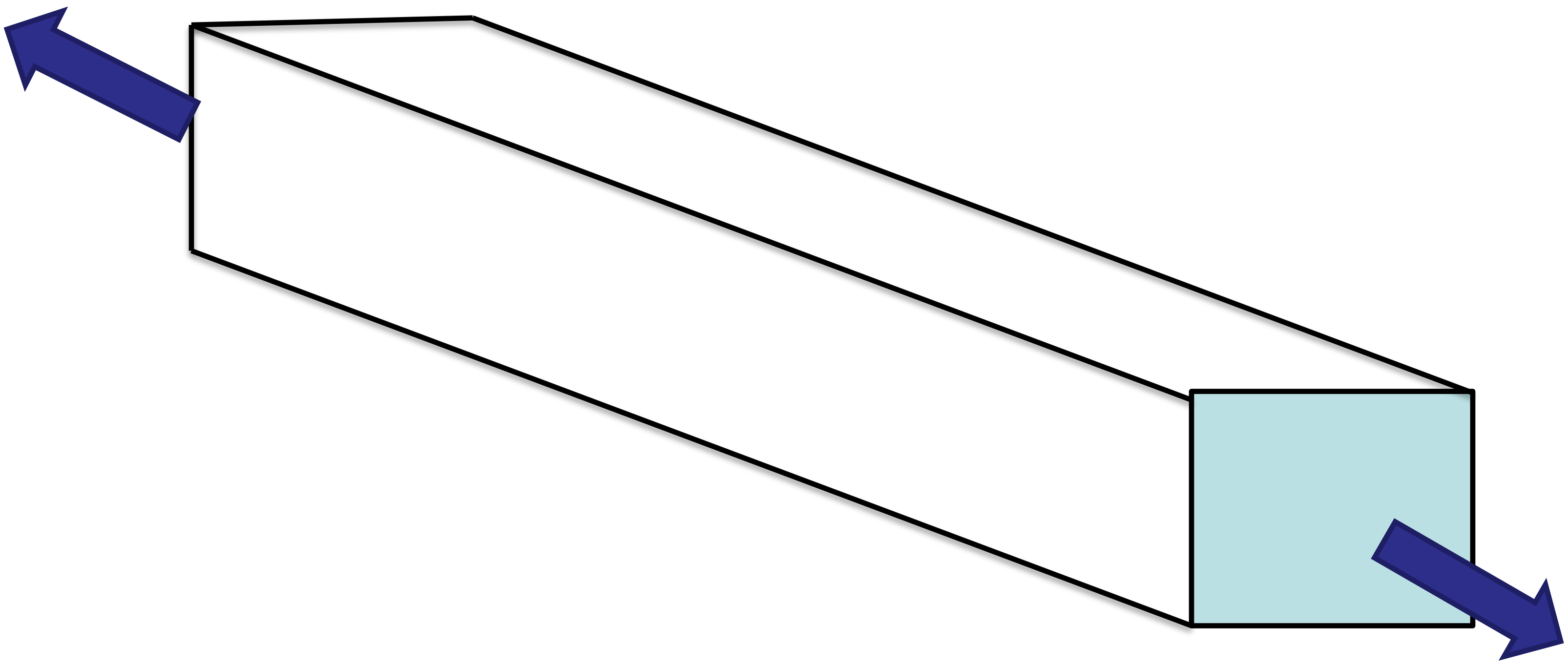
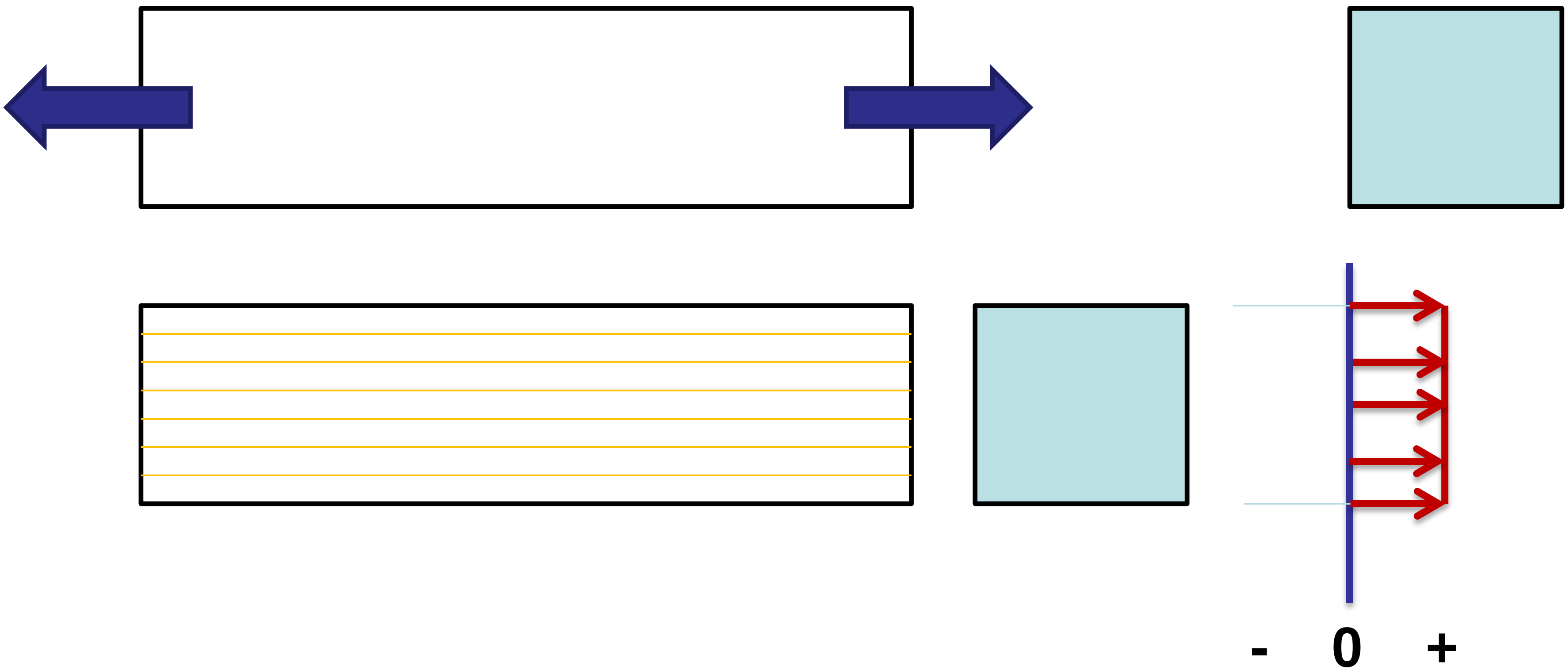
Stress distribution over the cross section



Stress distribution over the cross section



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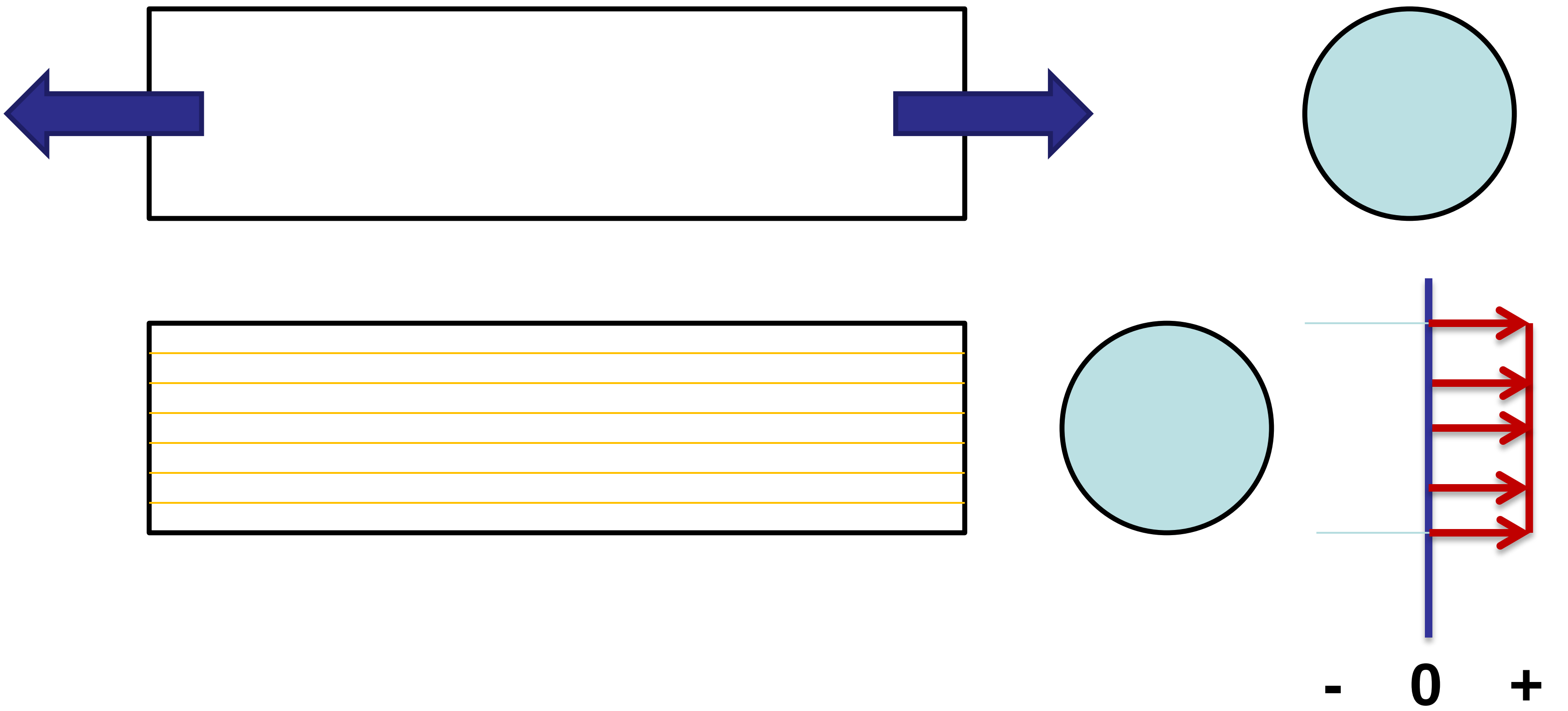
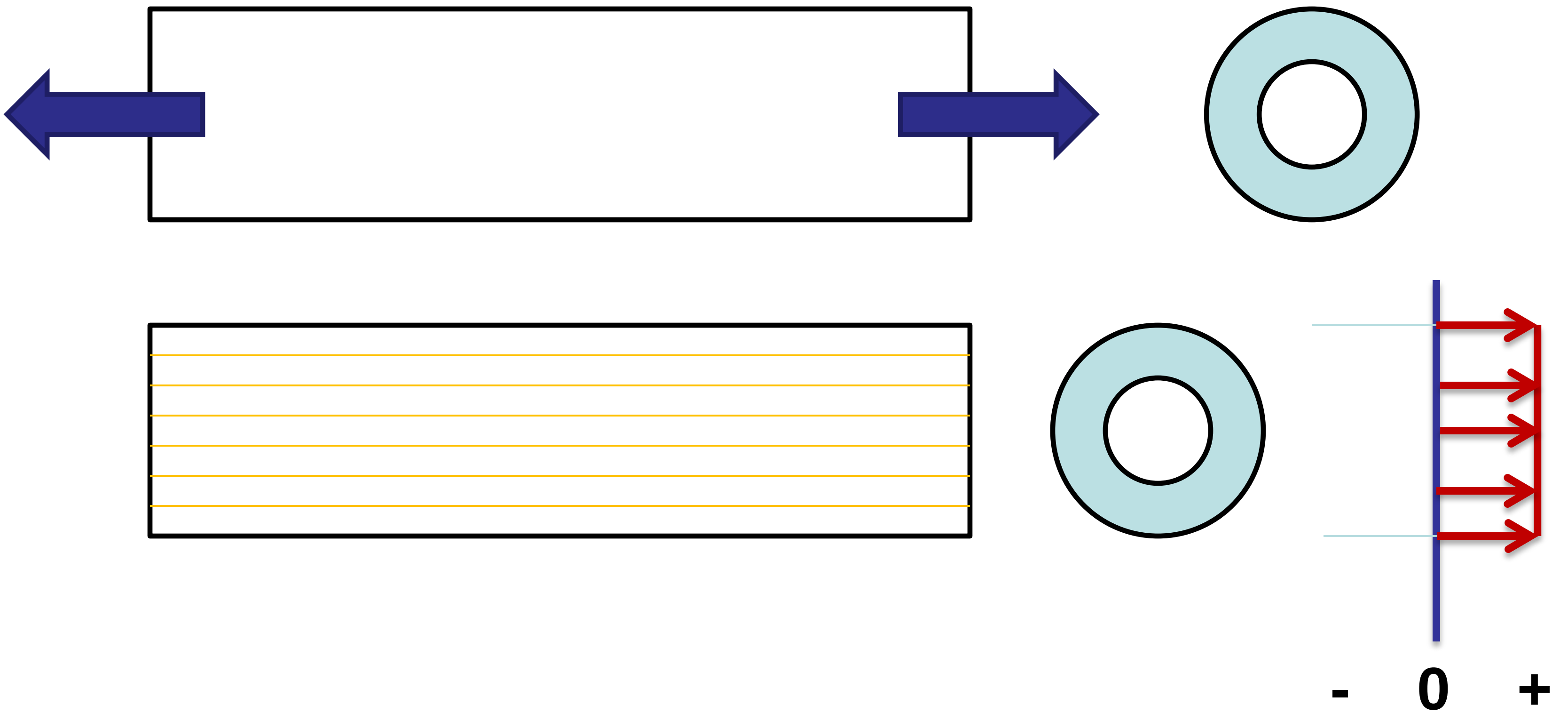
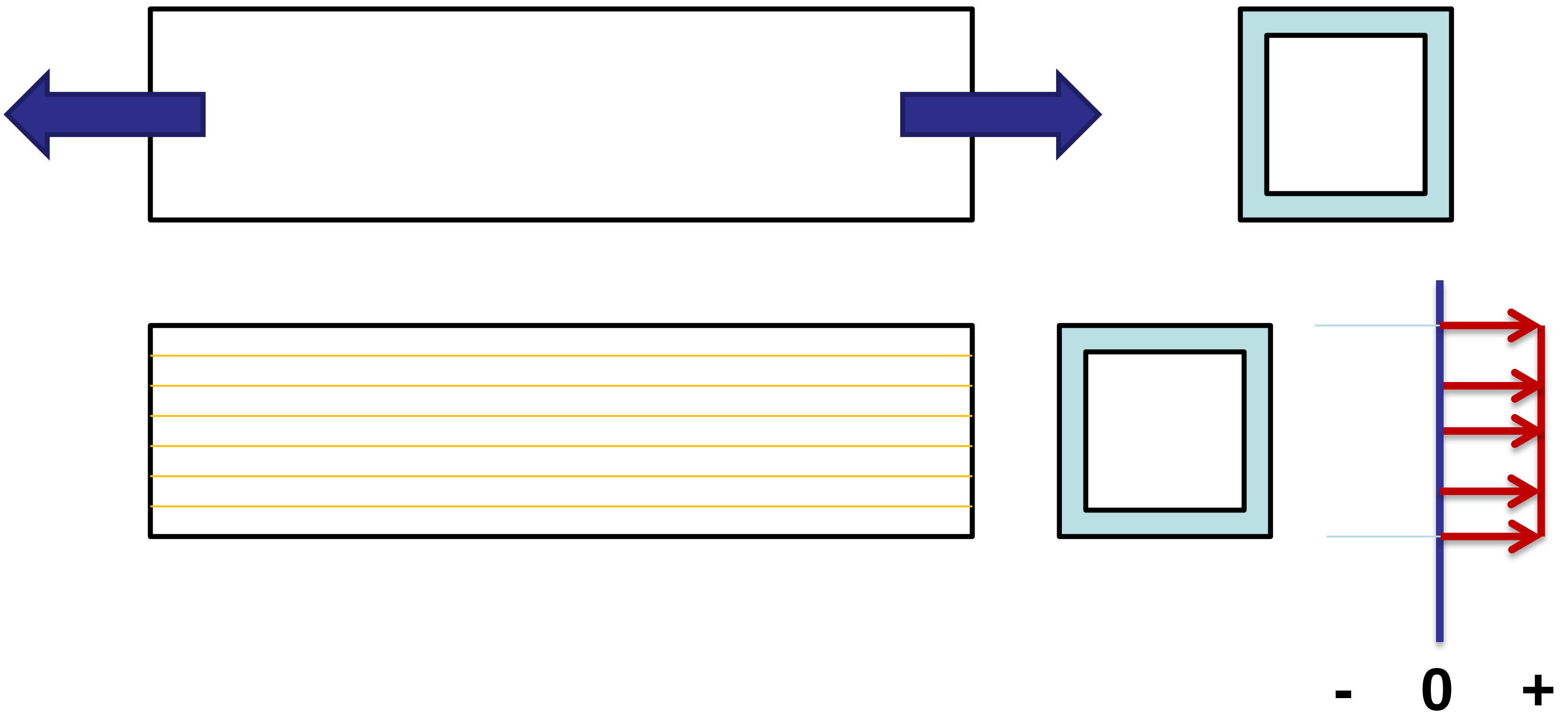


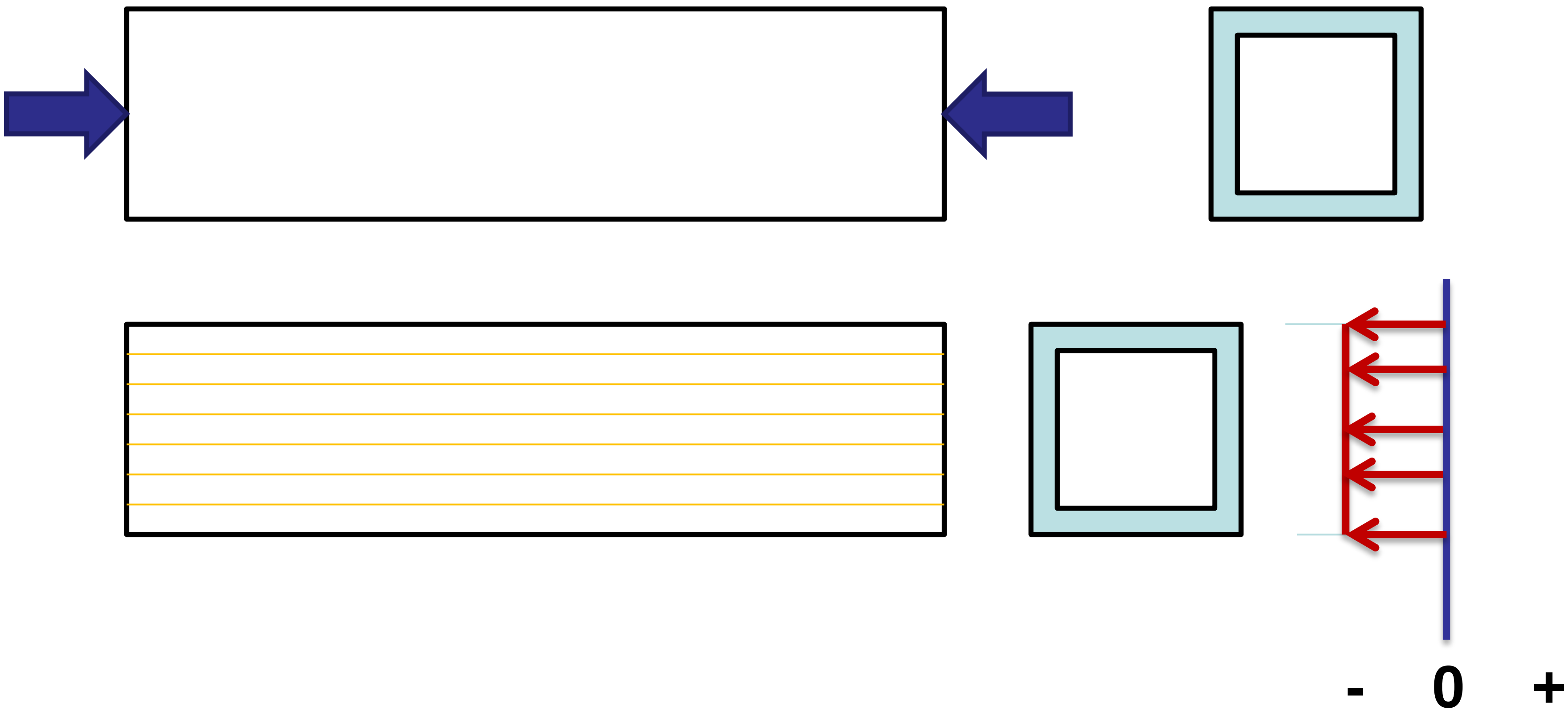
Stress distribution over the cross section



Stress distribution over the cross section



Stress distribution over the cross section



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Theory of Simple Bending

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Introduction

The behaviour of beams under bending is so complex.

The stress, strain, dimension, curvature, elasticity, are all related, under certain assumption, by *the theory of simple bending*. This theory relates to beam flexure resulting from couples applied to the beam without consideration of the shearing forces.

Notation

These notation are used in the next slides:

ε = strain

E = Young's Modulus = σ /e (N/m^2)

y = distance of surface from neutral surface (m).

ρ = Radius of neutral axis (m).

I = Moment of Inertia (m^4 - more normally cm^4)

Z = section modulus = I/y_{max}(m^3 - more normally cm^3)

F = Force (N)

x = Distance along beam

δ = deflection (m)

θ = Slope (radians)

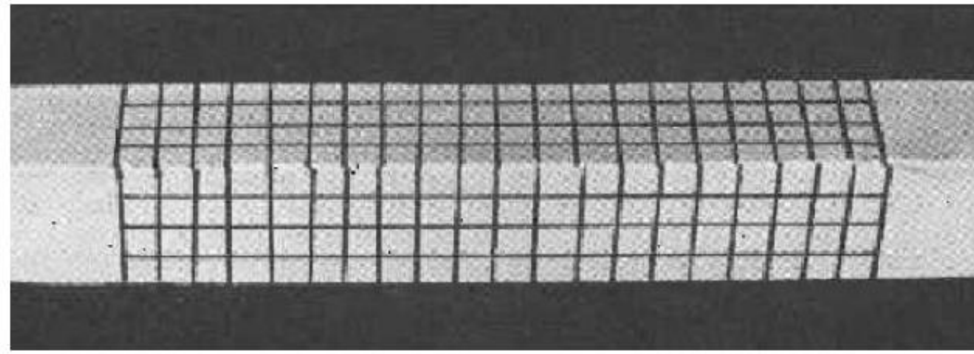
σ = stress (N/m^2)

Beam Bending Theory

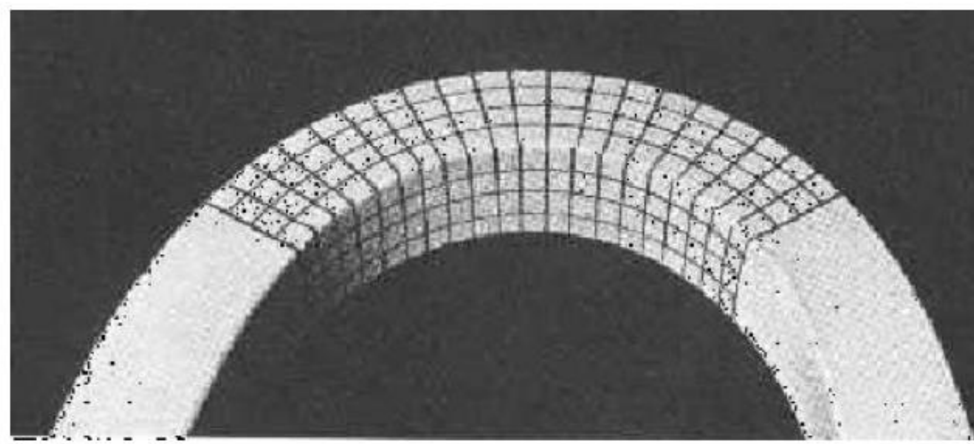
Consider the sponge beam shown:

Plane sections before bending were parallel to each other and perpendicular to the centre line of the beam.

Plane sections REMAIN plane and perpendicular to the centre line of the beam after bending.

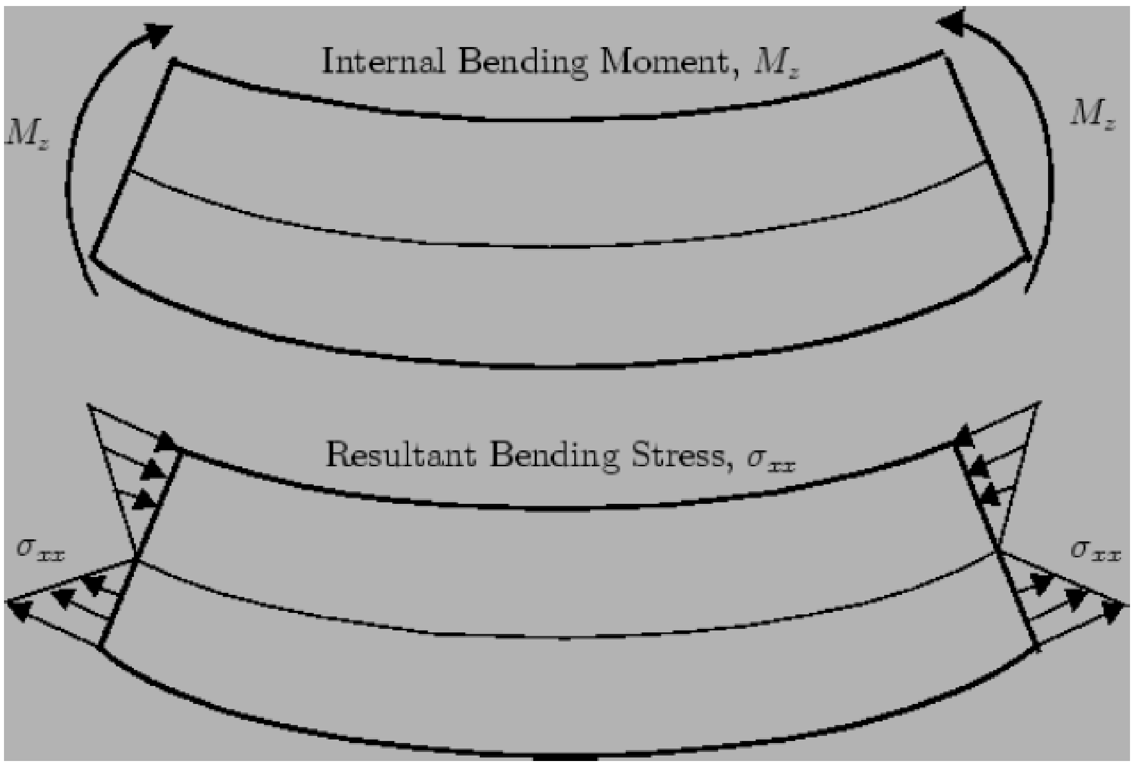


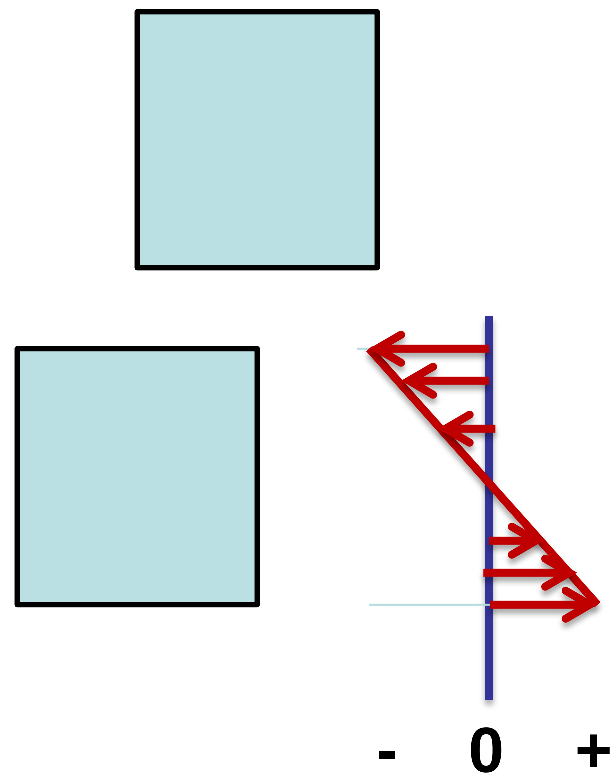
Before bending



After bending

Stress distribution over the cross section





Theory of Simple Bending

A straight bar of homogeneous material is subject to only a moment at one end and an equal and opposite moment at the other end.

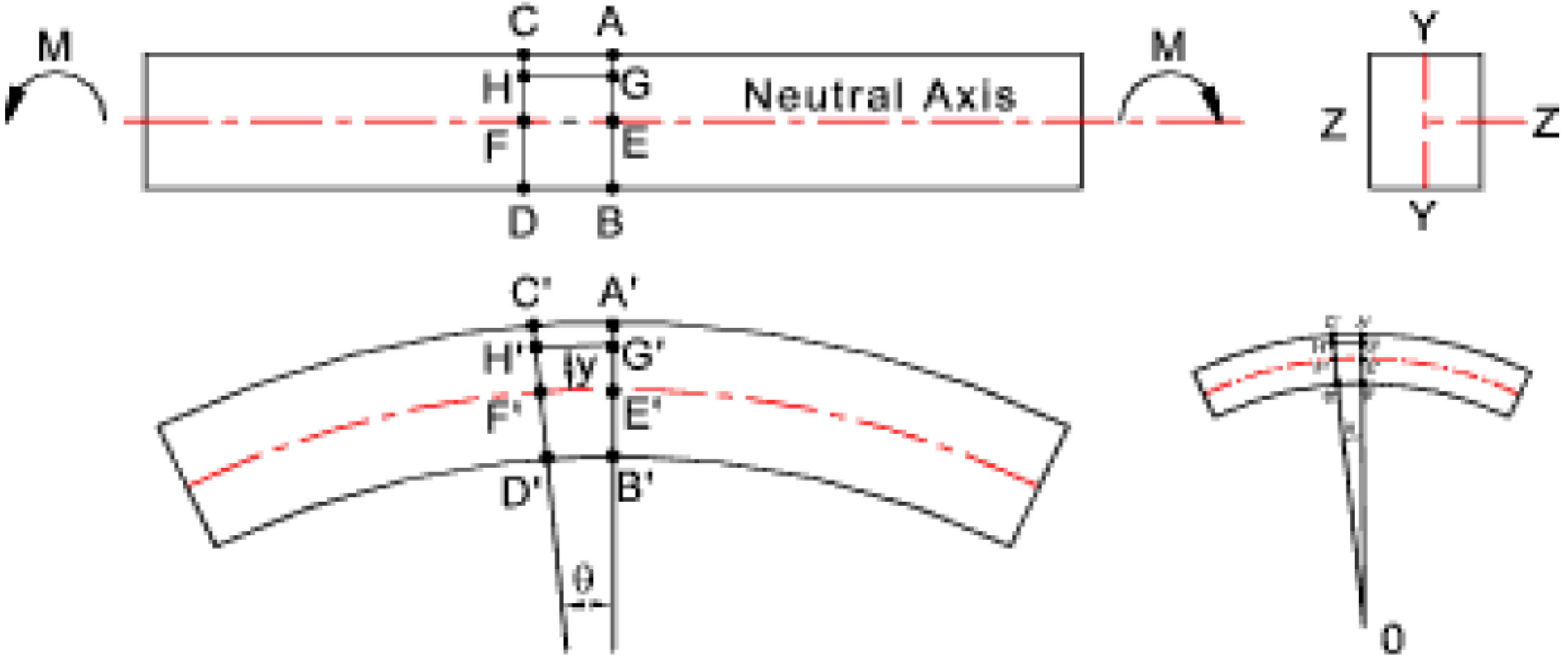
Assumptions

1.The beam is symmetrical about Y-Y.

2.The traverse plane sections remain plane and normal to the longitudinal fibers after bending (Beroulli's assumption).

3.The fixed relationship between stress and strain (Young's Modulus)for the beam material is the same for tension and compression ($\sigma = E.e$).

Theory of Simple Bending



Strains in Beams

1. Consider two section very close together (AB and CD).
2. After bending the sections will be at A'B' and C'D' and are no longer parallel.
3. AC will have extended to A'C' and BD will have compressed to B'D'.
4. The line EF will be located such that it will not change in length.
5. This surface is called neutral surface and its intersection with Z_Z is called the neutral axis.
6. The development lines of A'B' and C'D' intersect at a point 0 at an angle of θ radians and the radius of E'F' = ρ
7. Let y be the distance(E'G') of any layer H'G' originally parallel to EF.

$$H'G'/E'F' = (\rho + y)\theta / R\,\theta = (\rho + y)/\rho$$

8. And the strain ε at layer H'G' =

$$\varepsilon = (H'G' - HG) / HG = (H'G' - HG) / EF =$$

$$\varepsilon = [(\rho + y)\theta - \rho\,\theta] / \rho\,\theta$$

$$\varepsilon = \boldsymbol{y / \rho}$$

Stresses in Beams

The accepted relationship between stress and strain is

$$\sigma = E.\varepsilon$$

Therefore,

$$\sigma = E.\varepsilon = E.\ y/\rho$$

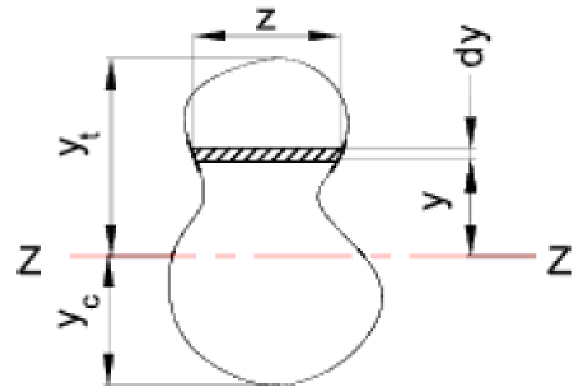
$$\sigma / E = y / \rho$$

$$\sigma = E.y / \rho \qquad or \qquad \underline{\boldsymbol{\sigma / y = E / \rho}}$$

Therefore, for the illustrated example, the tensile stress is directly related to the distance below the neutral axis. The compressive stress is also directly related to the distance above the neutral axis.
Assuming E is the same for compression and tension the relationship is the same.

Neutral axis

As the beam is in static equilibrium and is only subject to moments (no vertical shear forces) the forces across the section (AB) are entirely longitudinal and the total compressive forces must balance the total tensile forces.



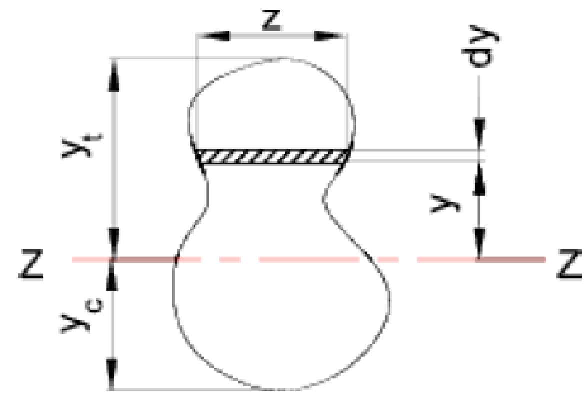
The internal couple resulting from the sum of (σ.dA .y) over the whole section must equal the externally applied moment.

$$\sum(\sigma.\delta A) = 0 \quad \text{therefore} \quad \sum(\sigma.z\ \delta y) = 0$$

$$\text{As} \quad \sigma = \frac{yE}{R} \quad \text{therefore} \quad \frac{E}{R}\sum(y.\ \delta A) = 0 \quad \text{and} \quad \frac{E}{R}\sum(y.\ z\delta y) = 0$$

This can only be correct if Σ(yδa) or Σ(y.z.δy) is the ***moment of area of the section*** about the ***neutral axis***. This can only be zero if the axis passes through the centre of gravity (***centroid***) of the section.

Bending Moment "Elastic Case"



The internal couple resulting from the sum of (σ.dA .y) over the whole section must equal the externally applied moment.

Therefore the couple of the force resulting from the stress on each area when totaled over the whole area will equal the applied moment.

The force on each area element = $\sigma.\ \delta A\ =\ \sigma.\ z.\ \delta y$

The resulting moment = $y.\sigma.\ \delta A\ =\ \sigma.\ z.\ y.\ \delta y$

The total moment $M = \sum(y.\sigma.\delta A\ )$ and $\sum\ (\sigma.z.y\ \delta y)$

Using $\frac{E}{R} y = \sigma$

$M = \frac{E}{R}\sum(y^2.\ \delta A\ )$ and $\frac{E}{R}\sum\ (z.y^2 \delta y)$

$\sum(y^2.\delta A\ )$ is the Moment of Inertia of the section(I)

From the above the following important simple beam bending relationship results

$$\frac{M}{I} = \frac{E}{R} = \frac{\sigma}{y}$$

Bending Moment "Elastic Case"

$$\frac{M}{I} = \frac{E}{\rho} = \frac{\sigma}{y}$$

M – Bending moment or Moment may vary depending on the load example

I – Moment of Inertia.

$I = bd^3/12$ for rectangular section and $y = d/2$

$I = \pi(D^4_o - D^4_i)/64$ for hollow pipe and $y = D_o/2$

σ – Stress due to bending moment.

E – Modulus of Elasticity or Young's modulus.

ρ- Radius of curvature due to bending.

y – distance measured from section centroid

Bending Stress “Elastic –Plastic Case”

The stress distribution beyond elastic limit may not be linear since it follows the stress strain curve under either tensile or compressive stresses respectively. In addition, the relation between stresses and strains may not be linear as well. There is no definite relation for bending stresses beyond yielding up until this time. The bending stresses should be calculated on a case-by case basis.

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Bending test

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Beam Bending Test

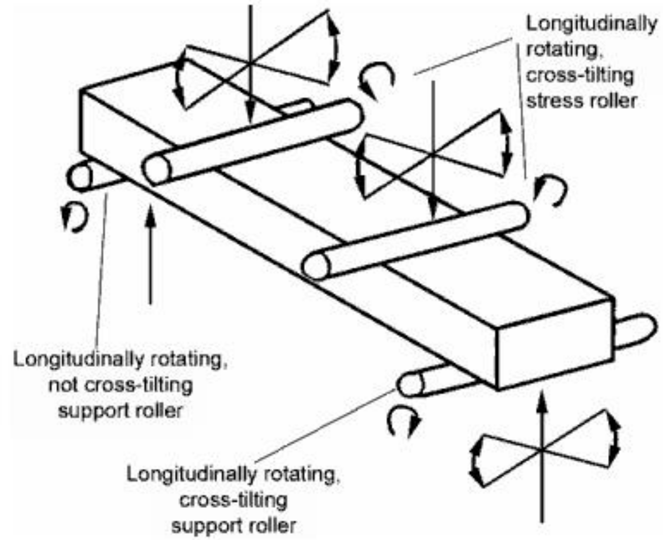
1. Extremely low ductility "brittle materials" does not allow measuring their mechanical properties (specially strength and stiffness) accurately by conventional tensile test, which is widely used for metals.
2. Brittle Materials, including cast iron, concrete, wood, ceramics, are tested Flexure Test by (Transverse Beam Bending Test).
3. There are two standard Flexure Test methods:

3-point Flexure Test **or** **4-point Flexure Test**

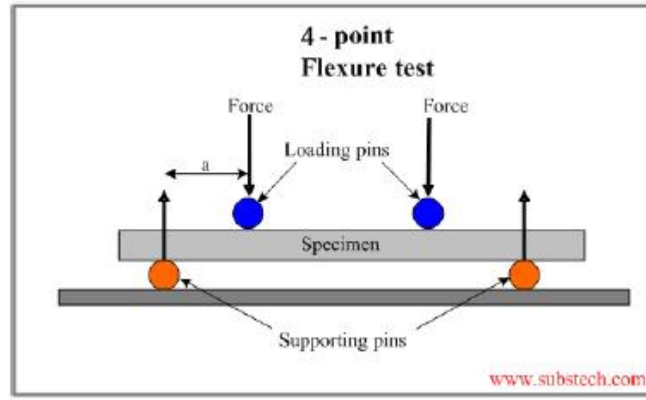
In this test a specimen with round, rectangular or flat cross-section is placed on two parallel supporting pins as follows;

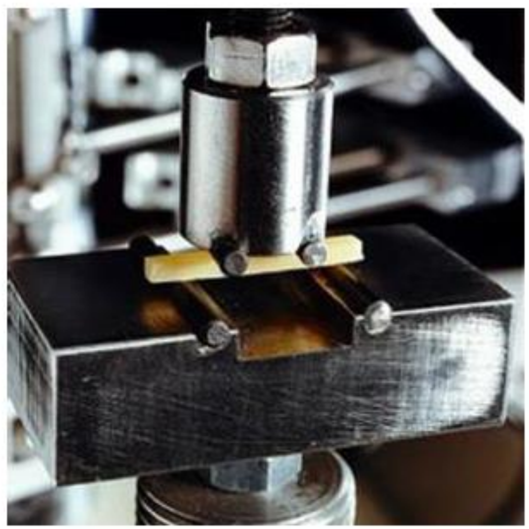
1. The supporting and loading pins are mounted in a way, allowing their free rotation about:
2. axis parallel to the pin axis;
3. axis parallel to the specimen axis.
4. This configuration provides uniform loading of the specimen and prevents friction between the specimen and the supporting pins.

4-point Flexure Test



In this test the loading force is applied by means of two loading pins with a distance between them equal to one-third of the distance between the supporting pins (span). The test provides at least the middle half of the specimen between the applied loads to be subjected to constant bending moment and zero shear. Yet the test is slightly more complicated.

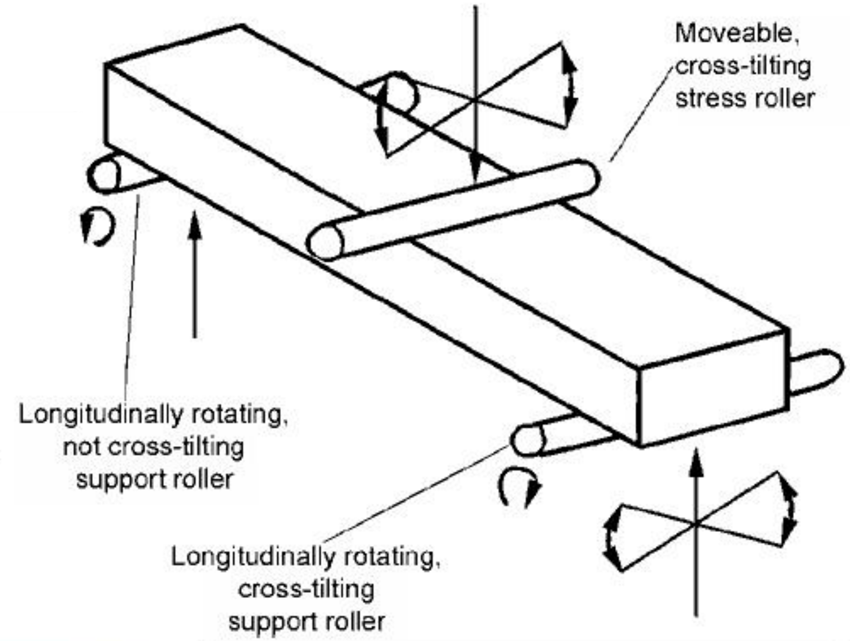


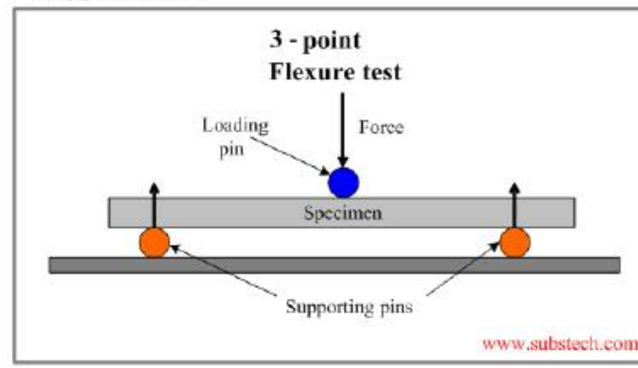


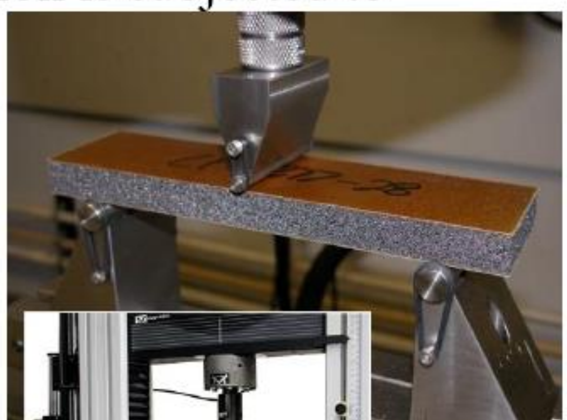
3-point Flexure Test

The loading force is applied in the middle (mid-span) by means of single loading pin.

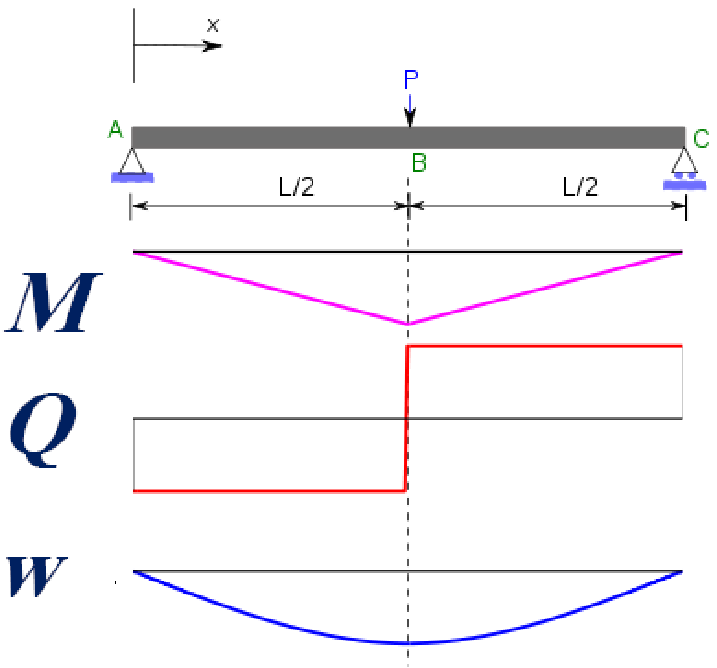
The test is much simpler to be carried out. Yet, only one single section is subjected to maximum bending moment in the whole specimen.



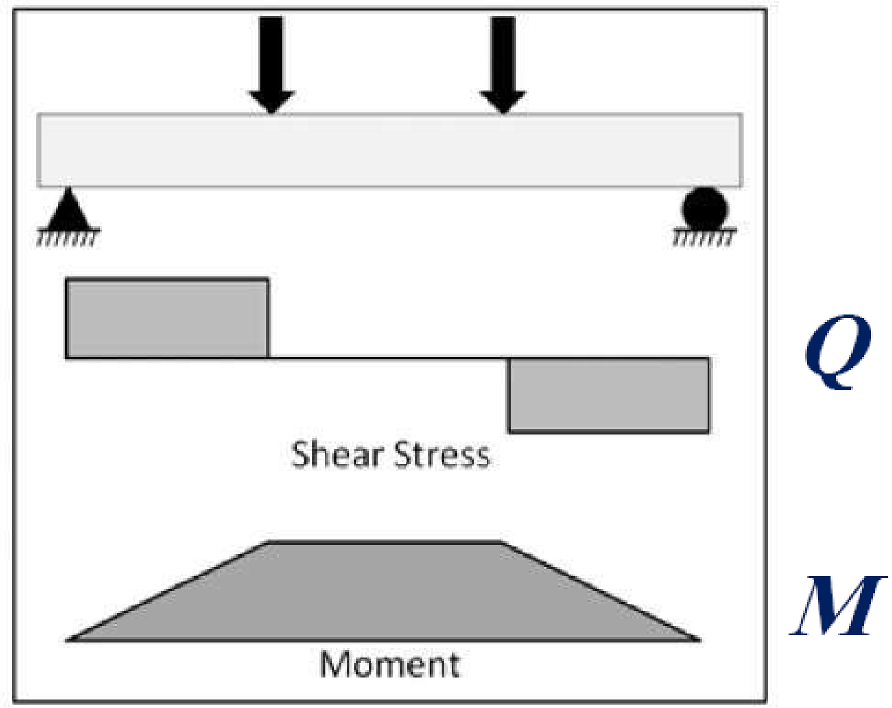




3-point Flexure Test



4-point Flexure Test



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The mechanical properties after bending test

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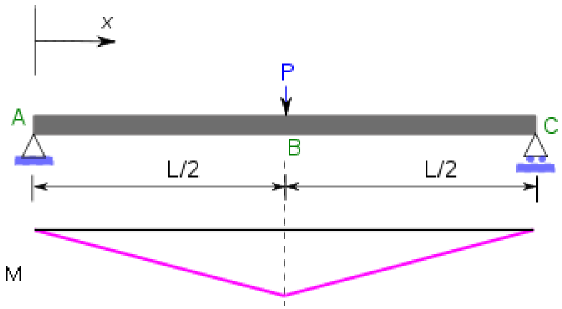
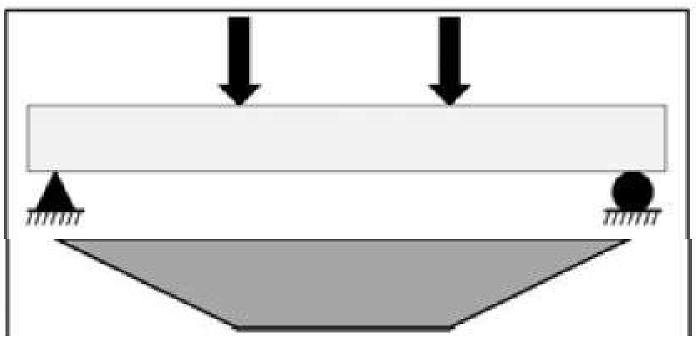
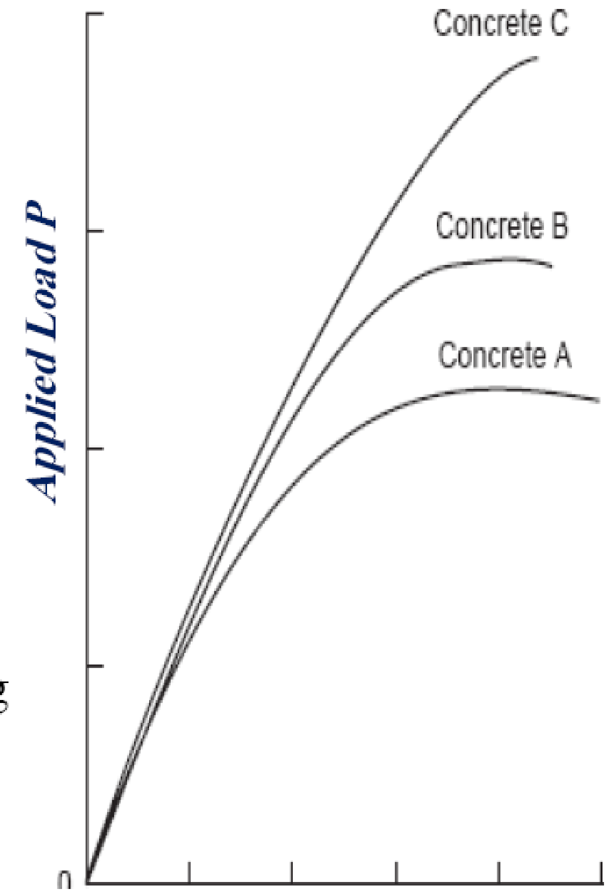
Mechanical Properties in Flexure





Dial Strain Gauge is used to measure the mid span deflection δ



Mid span deflection δ

As a result of the loading, the specimen bends, causing deformation, tension stresses in its convex side and compression stress in the concave side.
The applied load and the mid span deflection are measured for every load value. The results are then plotted in the load-deflection diagram.

Proportional Limit

Proportional limit is the maximum stress where stress is proportional to the strain.

Proportional limit load is defined as the end of the straight line. Thus, if the proportional limit load is P_{PL} and the original cross sectional area = Ao, then:

Maximum bending moment @ Prolortional limit = $\boldsymbol{M_{PL}}$

Proportional Limit = $\sigma_{PL} = \boldsymbol{M_{PL} * Y_{max} / I_x}$

$M_{PL} = P_{PL} * L/4$ *(for 3-point beam bending test)*

Y_{max} = h/2 for rectangular or square cross sections.

= $D_o/2$ for circular cross sections.

I_x = $bh^3/12$ for rectangular section

I_x = $\pi(D^4_o - D^4_i)/64$ for hollow pipe.

Stiffness "Young's Modulus"

The load deformation diagram for most engineering materials exhibit a linear relationship between applied load and deformation within the elastic region. Consequently, an increase in stress causes a proportionate increase in strain. The relation between *the modulus of elasticity or Young's modulus*, Applied load, measured deflection and the beam property is give as follows;

$$\delta_{PL} = \frac{P_{PL}L^3}{48EI}$$

Modulus of Rupture

Modulus of Rupture (Flexural Strength) is the stress of the extreme fiber of a specimen at its failure in the Flexure Test.

Modulus of rupture is calculated using the same formulae for simplicity and since there is no exact solution until now :

Maximum bending moment @ maximum load = $\boldsymbol{M_{max}}$

Proportional Limit = MoR = $\boldsymbol{M_{max} * Y_{max} / I_x}$

$M_{max} = P_{max} * L/4$ *(for 3-point beam bending test)*

Y_{max} = $h/2$ for rectangular or square cross sections.

= $D_o/2$ for circular cross sections.

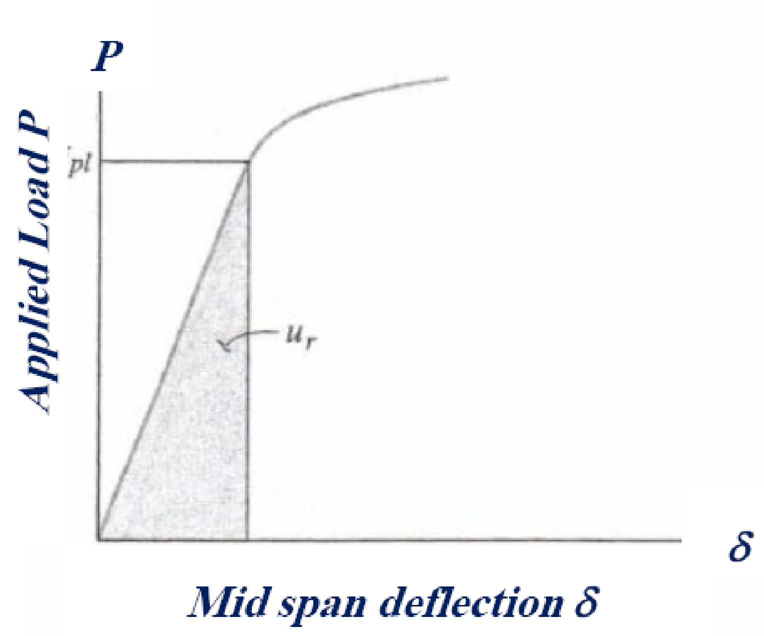
I_x = $bh^3/12$ for rectangular section

I_x = $\pi(D^4_o - D^4_i)/64$ for hollow pipe.

Modulus of Resilience

A material's resilience represents the ability of the material to absorb energy without any permanent damage to the material. In particular, when the load a reaches the proportional limit, the strain-energy density, is calculated by and is referred to as the *modulus of resilience Ur*. Mathematically it is the area under the straight line "elastic region" of the load-deformation curve per unit volume.

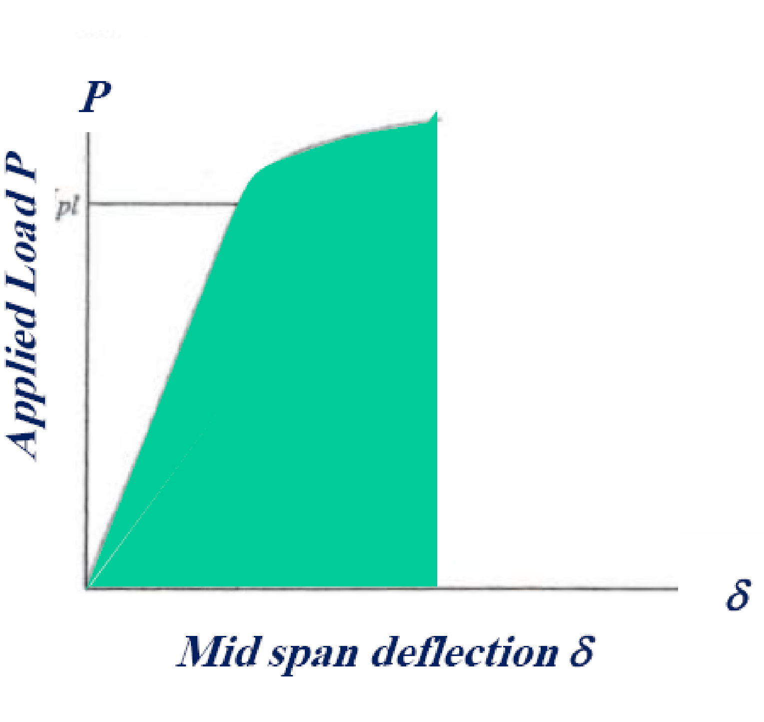
$$U_r = \frac{P_{PL} * \delta_{PL}}{2 * A * L}$$



Modulus of Toughness

Another important property of a material is *the modulus of toughness*, Ut. This quantity represents the entire area under the stress-strain diagram, and therefore it indicates the strain energy density of the material just before it fractures.

$$U_t = \frac{2 * P_{max} * \delta_{max}}{3 * A * L}$$



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Cold Bent Test

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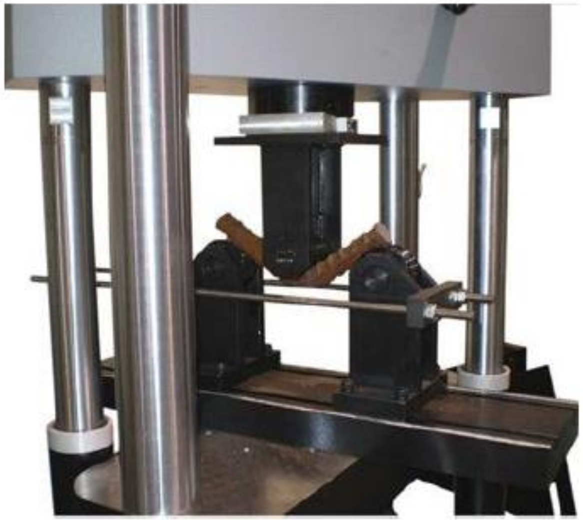
Cold Bent Test

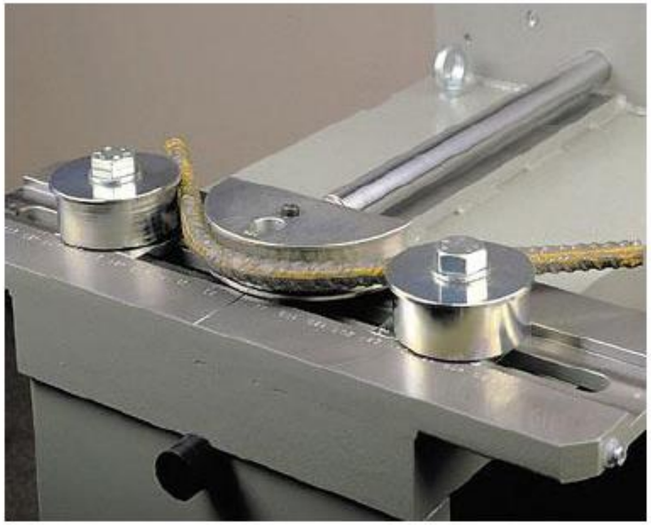
Rebar is bent into a multitude of different shapes to reinforce concrete structures. To ensure that the material is capable of being bent without significant ***strength*** loss cold bend test is used as a quality control check to ensure the bar's formability and the ***existence of ductility***. This test typically requires the sample to be bent around a forming pin to 180° angles, and visually inspected for development of any surface cracks. International standards specify requirements for the radius of the forming pin R, as a function of the bar diameter d_o as follows;

$$R = d_o \quad d_o < 25mm,$$
$$R = 1.5 * d_o \quad d_o < 25mm,$$

In addition, most rebar standards require that the bend test be completed in one continuous test stroke.

Cold Bent Test





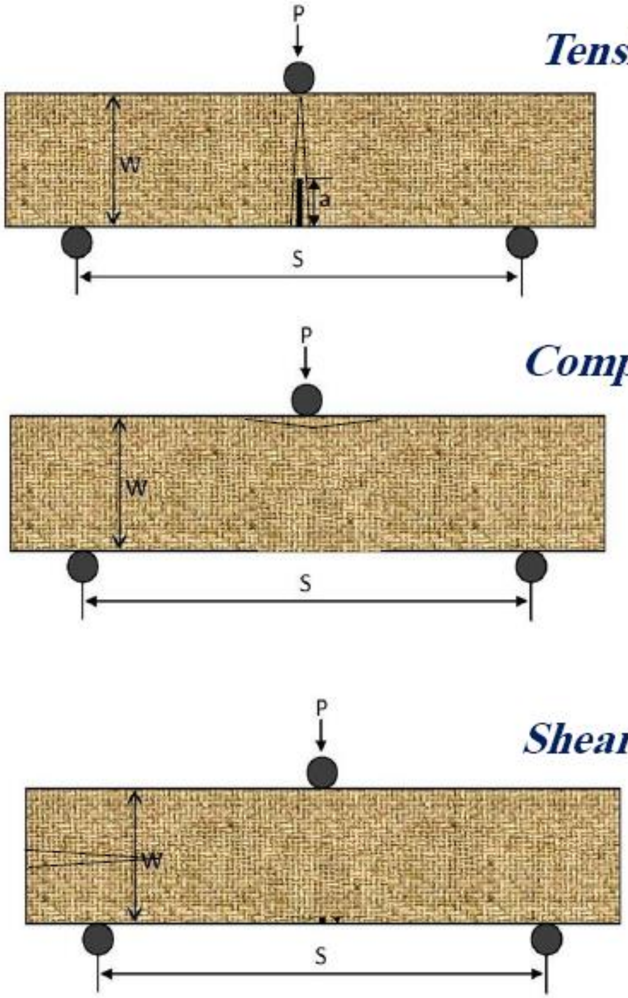
Mode of Failure

Tension
Lack of ductility

Compression

Shear
Small pin radius

Cold Bent Test



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Transverse shear stress

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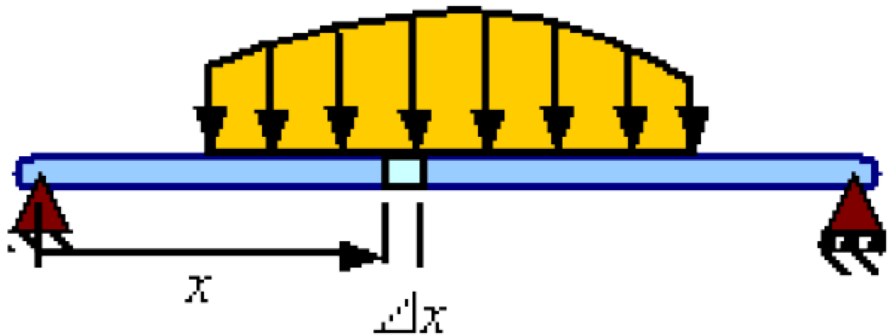
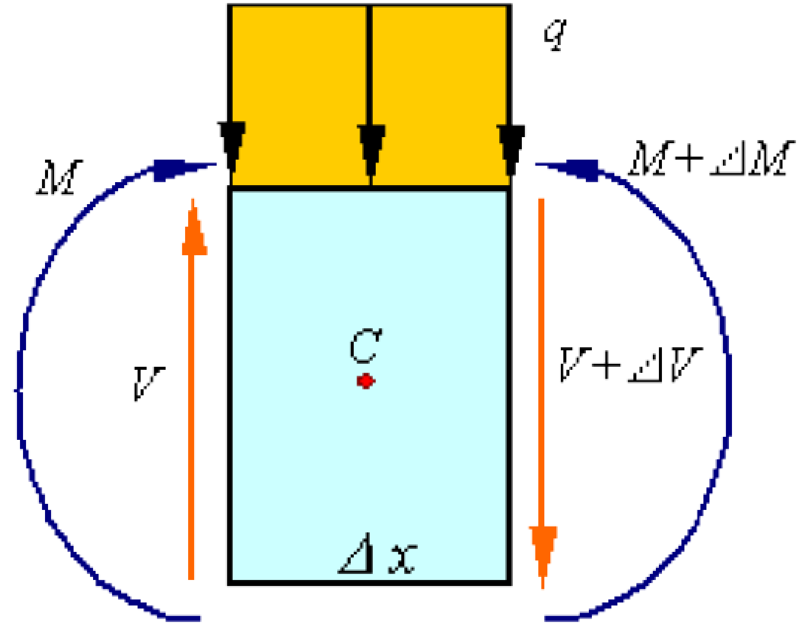
Flexure Shear

Flexure shear is produced due to the action of shearing force accompanying the bending of the beams under transverse loads.



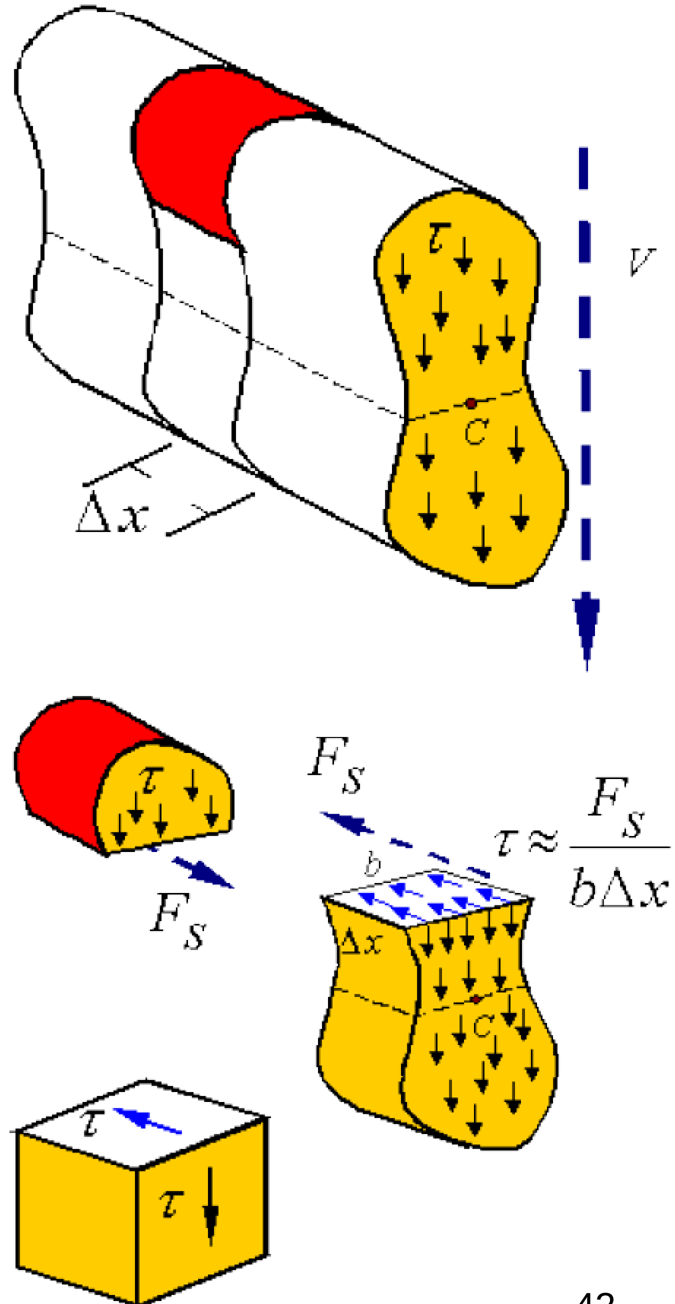
Consider a segment of the beam shown. The shear load on the vertical surfaces are generated by shear stress that can be calculated by the following process.



Flexure Shear Stresses

To calculate the shear stress τ generated from the shear load V consider removing the segment of the beam shown in red.

By symmetry of stress, shear stresses on the cross section results in equal shear stresses on the plane perpendicular to the cross section as shown. This shear stress results in a shear load F_s.



Flexure Shear Stresses

Therefore, equilibrium in the axial direction for this segment is written as:

$$F_s + \int_{A^*} [-\frac{(M + \Delta M)y}{I}]dA - \int_{A^*} [-\frac{My}{I}]dA = 0$$

$$\Rightarrow \quad F_s = \frac{\Delta M}{I} \int_{A^*} y dA$$

The integral in this expression is the first moment of the area A^* about the neutral axis. This first moment will be denoted by Q so that:

$$Q = \int_{A^*} y dA$$

Flexure Shear Stresses

The shear stress can now be calculated from the shear load by dividing it by the area it is applied on to get

$$\tau = \frac{F_s}{b\Delta x} = \frac{\Delta M}{\Delta x}\frac{Q}{Ib}$$

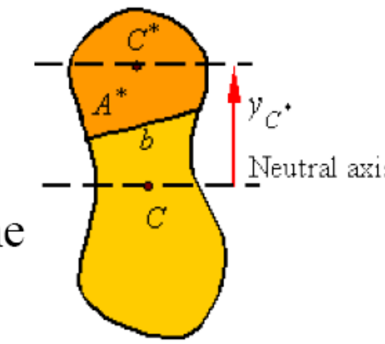
Taking the limit as $\Delta x \to 0$ gives $\tau = \frac{VQ}{Ib}$

where we note that $\frac{dM}{dx} = V$

Calculating the first moment of the area Q

The first moment of the area can be calculated from the relation

$$Q = A^* y_{C^*}$$

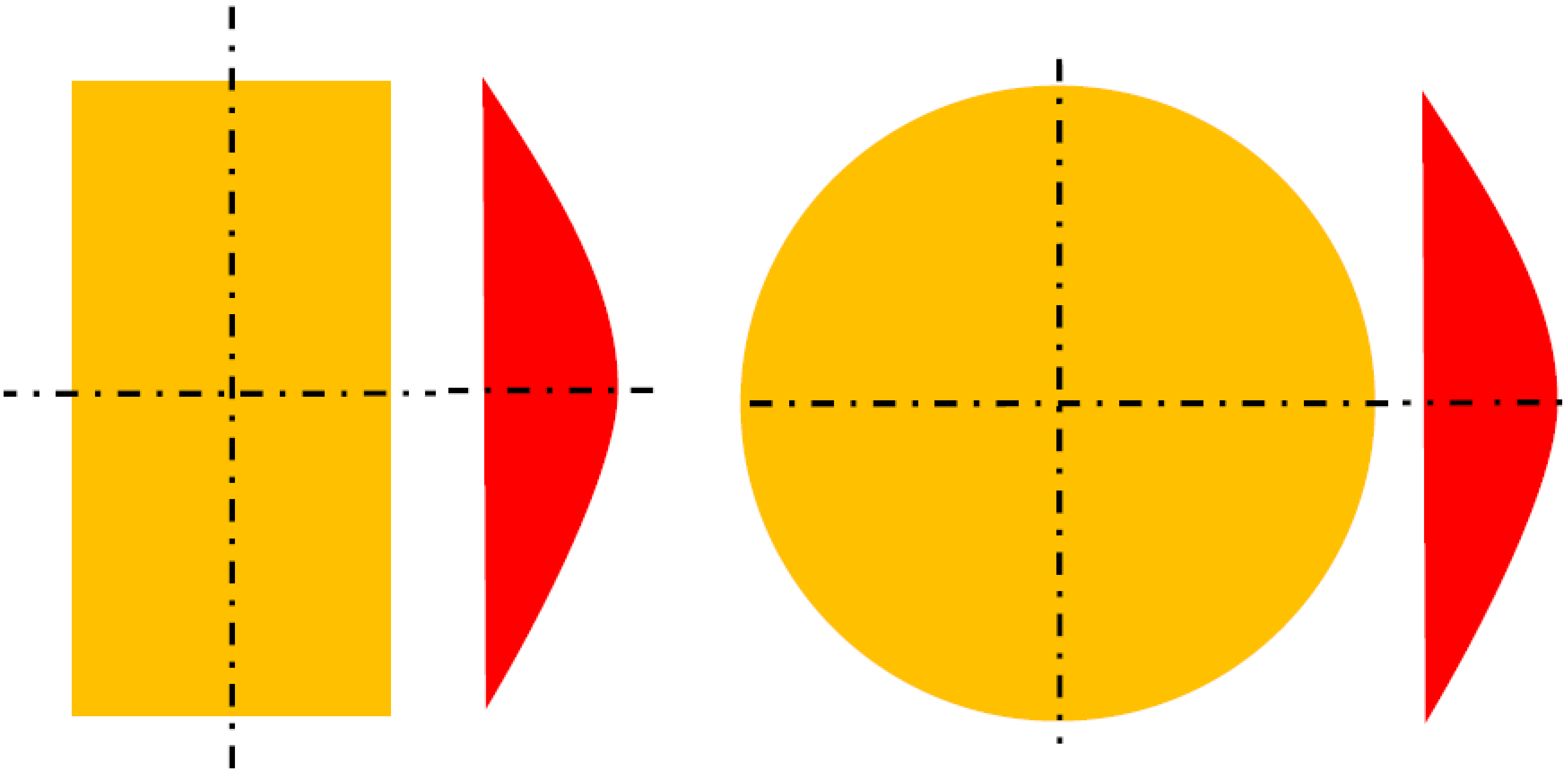


where A^* is the area of the part of the cross section that is considered, y_{C^*} is the vertical distance from the centroid of the cross section to the centroid of A^*.

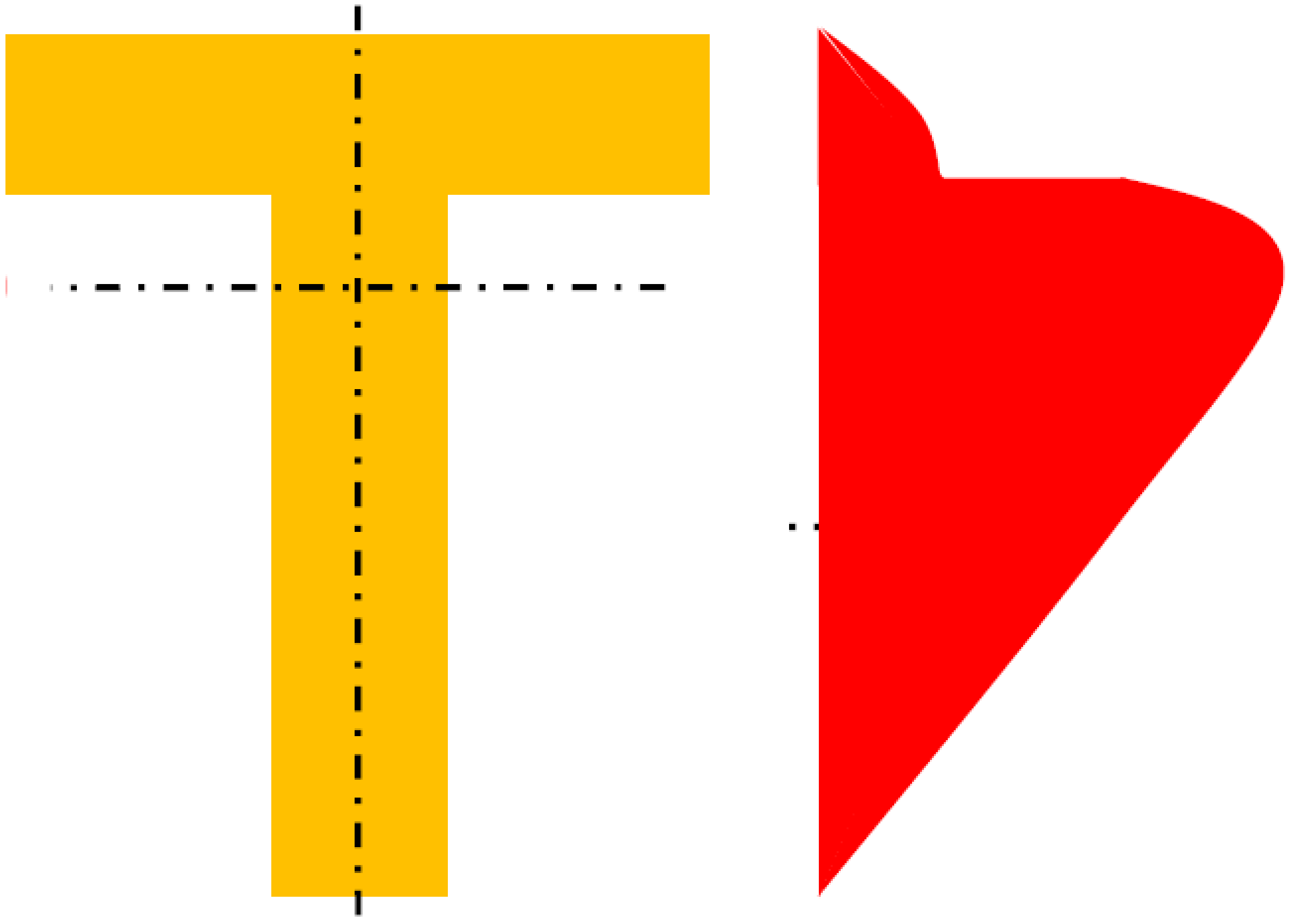
For composite areas, the first moment of area can be calculated for each part and then added together. The equation for Q in this case is

$$Q = \sum_{i=1}^{n} A_i^* y_{c_i^*}$$

Distribution of flexure shear stresses in standard sections



Distribution of flexure shear stresses in standard sections



Distribution of flexure shear stresses in standard sections